Tomographic inversion of 2-D WARP data based on Tikhonov regularization

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Summary

An algorithm for tomographic inversion of 2-D WARP data has been developed. Its purpose is the calculation of interface configurations and velocity distributions in layers, based on picked traveltimes of refracted and primarily reflected P-waves. An essential feature of our algorithm is the application of the Tikhonov regularization, which requires the resulting model to be as smooth as possible. Thanks to this feature, the inversion process is stable for any model parametrization, and realistic results may be obtained even if some model parameters are poorly controlled by the data.

Introduction

The traditional approach to WARP (Wide Aperture Reflection/Refraction Profiling) data interpretation is based on a trial-and-error method, which requires numerous calculations of seismic rays and traveltimes (Makris and Thiessen, 1984; Makris, 1995). In order to accelerate the process of interpretation, it was proposed to make use of the tomographic inversion (Huang et al., 1986; Zelt and Smith, 1992). But until now this approach has not been in common use. One of the possible reasons for that is unstable work of proposed inversion algorithms, caused by improper regularization. The authors did not go beyond the ordinary damped-least-square approach, which requires that each model parameter must be well controlled by the data.

The purpose of our study was to develop a more flexible algorithm, meeting the following requirements:

- It must be stable for any parameterization of the medium;
- Model parameters controlled by the data must be determined so as to minimize differences between observed and calculated traveltimes;
- Model parameters poorly or not controlled by the data must be determined from other evident considerations such as smoothness of the model and similarity of neighbour interfaces.

The algorithm

Data. The input information is picked traveltimes of refracted (turning) or primarily refracted P-waves. In the present form, our program cannot utilize multiple reflections or conversions.

Model. The model of the medium is assumed to consist of several layers divided by interfaces. Each interface is described by an arbitrary number of points joined by straight segments. Distances between neighbouring points may vary, but any vertical line may cross the given interface only once. Each layer is characterized by its own distribution of P-wave velocities. The velocities are specified in the nodes of a rectangular grid, the bi-linear interpolation is used to determine the velocities in between. Each layer must lie completely inside the relevant grid. The aim of the inversion is to calculate the configuration of interfaces and velocitiy distributions in layers.

Ray-tracing. Our inversion algorithm is based on the ray-tracing procedure written by Cerveny and Psencik (1984).

Linearization. Relations between model parameters and traveltimes are non-linear. But application of the linearization approach allows one to solve the problem through a sequence of several linear inversion steps. On each step, the following quadratic form must be minimized:

$$\begin{aligned}\Phi(\mathbf{m}) = (\mathbf{A}\Delta\mathbf{m} - \Delta\mathbf{t})^T(\mathbf{A}\Delta\mathbf{m} - \Delta\mathbf{t}) + \\ + \alpha^{int}\sum_k \mathbf{R}_k^{int} + \alpha^{vel}\sum_l \mathbf{R}_l^{vel}\end{aligned} \qquad (1)$$

where A is the matrix of partial derivatives:

$$A_{ij} = \left.\frac{\partial t_i(\mathbf{m})}{\partial m_j}\right|_{m_j = m_j^{cur}} ,$$

$\mathbf{m} = (m_1, m_2, \ldots, m_M)$ is the vector of model parameters (i.e. of interface depths in points specifying interfaces, and of velocities in grid nodes); M the is total number of model parameters; $\Delta\mathbf{m} = \mathbf{m}^{pert} - \mathbf{m}^{cur}$, $\mathbf{m}^{cur}$ is the current model; $\mathbf{m}^{pert}$ is the perturbed model to be found, $\Delta\mathbf{t} = (t_1^{obs} - t_1^{calc}, \ldots t_N^{obs} - t_N^{calc})$ is the vector of traveltime residuals, t_i^{obs} are observed traveltimes, t_i^{calc} are traveltimes calculated for the current model, $\mathbf{R}_k^{int}$ is the regularizator for the interface number *k;* $\mathbf{R}_l^{vel}$ is the regularizator for the velocity distribution in the layer number *l* (the notion of regularizator will be discussed below); α_{int}, α_{vel} are regularization parameters; first and second summation symbols concern respectively all interfaces and all layers, parameters of which are to be calculated. Explicit expressions for partial derivatives $\partial t_i(\mathbf{m})/\partial m_j$ may be found in Bishop et al., 1985; Zelt and Smith, 1992.

To calculate the minimum of the expression (1), we use the classical conjugate gradient method (Hestenes and Stiefel, 1952).

Regularization

Regularization operator for an interface. The correct choice of regularization plays a large role in the inversion process, if information contained in data is not enough to calculate the solution uniquely. Following Tikhonov and Arsenin (1977), we determine the regularization as the requirement that the function to be found must be in some sease the smoothest one. To make the Following discussion more clear, we have illustrated the work of various regularizators with the following simple example. Let the initial model consist of two layers separated by the interface (fig-la)). Configuration of this interface is to be found by means of linear inversion; other parameters (configuration of top and bottom interfaces, velocities in both layers) are assumed to be known. The unknown interface is represented by 41 equally spaced nodes. It is supposed that three reflection events were registered For the unkaowa interface, the relevant two-way traveltimes indicating that the uakaowa interface must cross three points marked by asterisks. Configuration of the interface in other points is not controlled by the data and therefore depends on the regularization operator.

Damped least squares. The following Tiionov's regularizator corresponds to this widely used approach:

$$R^{int}[Z] = \int_{X_{left}}^{X_{right}} [Z(x)]^2 dx,$$

where $Z(x)$ is a function describing the configuration of a given interface; X_{left} and X_{right} are X-coordinates of the region boundaries. For simplicity, we will write all expressions in the integral form, though in fact their finite-difference analogues must be used. According to the damped-least-squares approach, unkaown values not controlled by the data are equal to zero. As one can see from fig-1b., for a case of insufficient data, this approach may produce a rather unreal resulting model.

Minimization of the first derivative:

$$R^{int}[Z] = \int_{X_{left}}^{X_{right}} \left(\frac{dZ(x)}{dx}\right)^2 dx$$

In our case, this approach results in the linear interpolation of the unknown function between the points, where the function values are determined by the data. The inversioa result looks much better thaa in the case of damped least squares (fig.1c), but the presence of points where the first derivative of the interface function possesses breaks looks undesirable. Therefore, the second derivative should also be included into the regularization operator.

Joint minimization of first and second derivatives. The Tikhonov's regularizator for this case is:

$$R^{int}[Z] = \int_{X_{left}}^{X_{right}} \left[\left(\frac{dZ(x)}{dx}\right)^2 + \rho^2 \left(\frac{d^2Z(x)}{dx^2}\right)^2\right] dx$$

The relative weight of each derivative is determined by the parameter ρ. One can show that the meaning of this parameter is a characteristic distance: the resulting function is practically linear in all points, which are remote from those data-controlled by more thaa ρ. We adjusted the value of parameter ρ as doubling the average distance between neighbour interface points. Application of this regularizator to our example results in a model which appears quite reasonable (fig.1d). The only feature one can

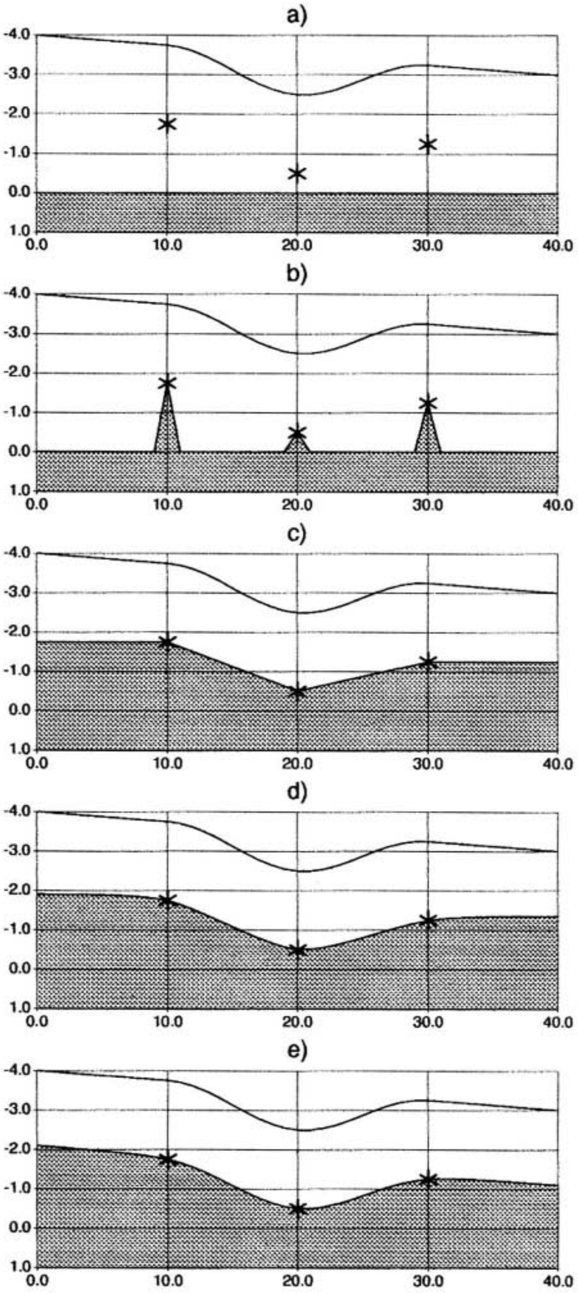

Figure 1: Calculation of the interface configuration by meaas of linear inversion with various Tiionov regularizators: (a) The initial model; (b) Damped least squares; (c) Minimization of the first derivative; (d) Joint minimization of first and second derivatives; (e) Modified joint minimization of first and second derivatives. Length units are arbitrary. Asterisks denote the points which are to be crossed by the interface.

find undesirable is the slope disagreement between the interface to be found and the one above. To improve this drawback, we constructed a regularizator introducing information of the previous interface.

Modified joint minimization of first and second derivatives:

$$R^{int}[Z] = \int_{X_{left}}^{X_{right}} \left[\left(\frac{d[Z(x) - Z_{prev}(x)]}{dx} \right)^2 + \right.$$

$$\left. + \rho^2 \left(\frac{d^2 Z(x)}{dx^2} \right)^2 \right] dx$$

where Z_{prev} (x) is the configuration of the interface above the one to be found. Thanks to the first term of the regularization operator, the interface being searched tries to mimic the configuration of the previous one; the second term adds smoothing. In our example, the interface calculated under the above written regularization condition appears very realistic (fig. fe). This regularization operator is used in our inversion algorithm.

Regularization operator for velocity distribution in a layer. The velocity regularizator used is similar to that proposed earlier for first arrival tomographic inversion (Ditmar, 1993; Ditmar et al., 1995). It includes only the terms depending on first derivatives and in the integral form may be expressed as:

$$R^{vel}[w] = \int_{\Omega} \left[10 \left(\frac{\partial w(x,z)}{\partial x} \right)^2 + \left(\frac{\partial w(x,z)}{\partial z} \right)^2 \right] dx\, dz$$

$$\text{where} \quad w(x,z) = \frac{V^{-1}(\mathrm{x}, z) - V_{cur}^{-1}(x,z)}{V_{cur}^{-1}(x,z)};$$

$V(x,z)$ is the velocity distribution to be found; $V_{cur}(x,z)$ is the velocity distribution, from which the linear inversion starts; Ω is the area covered by the velocity grid, which corresponds to the given layer. Our previous studies proved the stable work of this regularization condition. Naturally, the absence of the second derivative causes some smoothness discontinuities, but here they are not as evident as in the case of interfaces. Firstly, velocity functions are usually more smoothed, and secondly, visualization by means of isolines conceals smoothness discontinuities.

The optimal inversion strategy

Inversion: "layer-by-layer" versus "simultaneous". Our first numerical experiments demonstrated that "simultaneous" inversion strategy when all model parameters are calculated on every iteration, may fail. If the initial model is far from the true one, the problem to be solved is highly non-linear, and the very first iteration introduces into the model unreal features, such as layers of a very low velocity. Following iterations cannot considerably change this model. To cope with this problem, we attempted the "layer-by-layer" inversion strategy. Initially, only the velocity in the first layer and the configuration of the underlying interface are found iteratively. These parameters are then frozen; the parameters of the next layer are determined, and so on. Considerable reduction of the number of model parameters to be found simultaneously makes the inversion procedure much more stable.

On the other hand, the simultaneous inversion works properly, if the initial model is close to the true one. Thus, we finally found a combined inversion strategy as optimal. Initially, layer-by-layer inversion is performed (it is relativelly time-consuming, so only small amount of shots are considered on this step). The produced model is then used as the initial guess for simultaneous inversion when all data are utilized.

Regularization parameters. The smoothness of the model is controlled by regularization parameters, see expression (1). The more interface regularization parameter, the more smoothed interfaces in the calculated model; the same is true for velocities. There are no formal rules for optimal choice of these parameters; the interpreter may specify them, taking into account the following considerations:

- Regularization parameters must be gradually decreased during non-linear inversion.
- Relative weight of the velocity regularization parameter must normally be much more than that of the interface regularization parameter. It means that if a traveltime residual may equally be explained by interface variations and velocity variations, the most of residual reduction (say, 90%) must fall on interface variations.
- The non-linear inversion process must be stopped, if following iterations either result in unreal values of some model parameters or do not reduce the average traveltime residual,

Identifkation of arrivals. One of the most importaut point of interpretation is the correct identification of picked arrivals. Each arrival must be classified either as a reflection from a certain interface or as a refraction in a certain layer- Identification must be checked on every iteration and corrected, if necessary. Fortunately, these corrections usually do not influence the model to such an extent that the non-linear inversion must be repeated from the very beginning.

Application to real data

We have considered a set of seismic records preliminary interpreted by the trial-and-error method. The trial-and-error approach combined with some borehole information resulted in a model consisting of 6 sedimentary layers overlaying a high velocity basement (fig.2a). Thereafter, traveltimes were independently picked from the seismic records, To eliminate possible identification errors, the picked traveltimes were compared with traveltime curves calculated for the above-mentioned model and then classified. The result of following non-linear tomographic

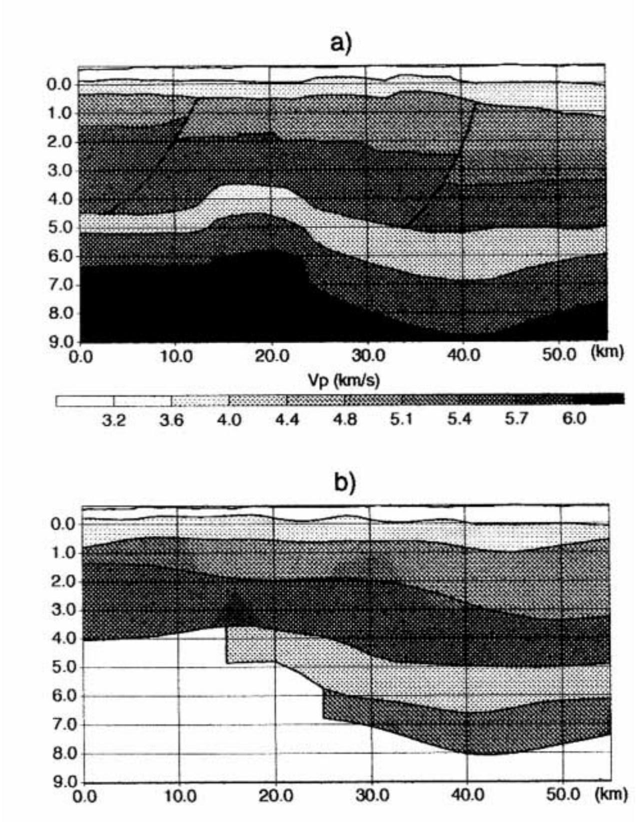

Figure 2: An example of real data interpretation. (a) The model obtained by means of trial-and-error approach combined with some borehole information; (b) The result of tomographic inversion (the area not controlled by the picked data is shown white).

inversion is shown in fig.2b. Parameters of the first three layers turned out to be controlled by the picked data quite well, and one can see a good agreement between top parts of the two models. Smoothnes of interfaces obtained by tomographic inversion is an inevitable consequence of our regulariaation condition combined with limited data accuracy; interface breaks must be introduced manually into the results of tomographic inversion if necessary. As for the lower part of the calculated model, the agreement is not so good, in spite of the fact that we froze the velocity distribution in the fifth (low-velocity) layer, based on general information about the geological structure (traveltime data alone will not find simultaneously velocity parameters and thickness of a low velocity layer). The disagreement, may be explained by the small number of picked data, controlling the lower part of the model. Nevertheless, the general behaviour of interfaces (a bump in the left part of the region, a trough in the right part) is restored correctly even for the lower part.

Conclusions

We have presented the first version of our algorithm for tomographic inversion of WARP data. We believe that its usage will make interpretation much faster. From our first experience, we would expect that one of the main problems an interpreter may face is the correct identification of seismic arrivals, and we intend to pay special attention to this problem. We also plan to provide our algorithm with the following capacities: 1) utilization of borehole information; 2) setting the points of interface breaks; 3) consideration of pitching-off layers.

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